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Self Similarity Solution Of Plane Shock Wave In A Medium With Variable DENSITY

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Abstract: *In this paper Self similar solutions for one dimensional flow of plane shock wave propagating into an uniform atmosphere with variable density and pressure, are obtained where the uniform atmosphere assumed to be at rest.*

I. Introduction

Ojha and Onkar [1] studied the propagation of shock waves in an inhomogeneous self gravitating gaseous mass in which the disturbances are headed by a shock of variable strength. Sakurai [2], Witham [3] studied the problem of spherical shock wave. Verma and Singh [4], Singh and Srivastava [5] have considered the problems of spherical shock waves in an exponentially increasing medium under the uniform pressure. Singh and Srivastava [6] have discussed the problems of magnetoradiative shock in a conducting plasma. Similarity solutions for shock waves phenomena in magnetogasdynamics have been obtained by number of authors eg Vishwakarma [7], Shilpa Shinde [8], Michaut Haut and et al. [9]. Srivastava and Litoria had found the solution

for propagation of plane shock wave in magnetogasdynamics [10],] Jitendra kumar soni have considered the problems of analysis of Self Similar Motion in the Theory of Stellar Explosion [11], recently

In the present paper a self similar model of the flow behind a plane shock wave has been considered in which we have assumed that the disturbance is headed by a shock surface of variable strength and is propagating into a medium with variable density and pressure. The shock waves propagate in a uniform atmosphere which is assumed to be at rest.

The shock position in this problem is given by

$$R = At^{\mu}, \tag{1.1}$$

where A and μ are constants and $\mu < 1$.

We assume that the density distribution is given by

$$\rho_0 = br^{\beta}, \tag{1.2}$$

where b and β are constants.

II. Equation Of Motion

The basic equations governing the motion of the fluid in plane symmetry are

$$\frac{\partial \rho}{\partial t} + u\,\frac{\partial \rho}{\partial r} + \rho\,\frac{\partial u}{\partial r} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{1}{\rho}\,\frac{\partial P}{\partial r} = 0, \tag{2.2}$$

$$\frac{\partial P}{\partial t} + u\,\frac{\partial P}{\partial r} - \frac{(\gamma - 1)P}{\rho}\left(\frac{\partial \rho}{\partial t} + u\,\frac{\partial \rho}{\partial r}\right) = 0; \tag{2.3}$$

where ρ, u, P, t *and* γ are the density, velocity, pressure, time and ratio of specific heat of the gas.

III. Similarity Solutions

We introduce the similarity variable

$$\eta = r/R(t), \tag{3.1}$$

and take the solution of the basic equations in the form,

$$u = \dot{R}\,V(\eta), \tag{3.2}$$

$$\rho = \rho_0\,D(\eta), \tag{3.3}$$

$$p = P_0\ \dot{R}^2\ P(\eta), \tag{3.4}$$

where V, D, P are functions of η only and $\dot{R} = dR/dt,$ the shock velocity.

IV. Solution Of Equations Of Motion

In view of similarity transformation (3.1) – (3.4), the basic equations (2.1)-(2.3) take the form

$$\eta\,(1-\eta)D' + (\beta + \eta V')D = 0, \tag{4.1}$$

$$\left(\frac{\mu-1}{\mu}\right)V + (V-\eta)V' + P' + \frac{\beta}{\eta}P = 0, \tag{4.2}$$

$$\left[2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)\beta}{\eta}\right]P + (V-\eta)P' - (1-\eta)(\gamma-1)\frac{PD'}{D} = 0. \tag{4.3}$$

V. The Jump Conditions

The jump conditions for a strong shock wave are

$$u_1 = \frac{2\dot{R}}{\gamma+1}, \tag{5.1}$$

$$P_1 = \frac{2\rho_0\ \dot{R}^2}{\gamma+1}, \tag{5.2}$$

$$\rho_1 = \frac{(\gamma-1)\rho_0}{(\gamma-1)}; \tag{5.3}$$

where suffix 1 denotes the values of flow variables immediately behind the shock front.

The system of equations (4.1) – (4.3) can be reduced to

$$D' = \frac{D\left[(V-\eta)\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}(V-\eta)^2 + \frac{\beta P}{\eta}(V-\eta) - \left\{2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)\beta}{\eta}\right\}P\right]}{(1-\eta)\left[(V-\eta)^2 - (\gamma-1)P\right]}, \tag{5.4}$$

$$V' = -\frac{\alpha}{\beta} - \left\{ \left[(V-\eta)\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}(V-\eta) - 2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)}{\eta}\beta \right] P \right\} / \left[(V-\eta)^2 - (\gamma-1)P \right]$$

(5.5)

$$P' = -\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}P + (V-\eta)\frac{\beta}{\eta} + (V-\eta)\left\{ \left[(V-\eta)\left(\frac{\mu-1}{\mu}\right)V - \frac{\beta}{\eta}(V-\eta)^2 + \frac{\beta P}{\eta}(V-\eta) \right.\right.$$

$$\left. - \left\{ 2\left(\frac{\mu-1}{\mu}\right) + \frac{\beta V}{\eta} - \frac{(\gamma-1)\beta}{\eta} \right\} \right] P \Big\} / \left[(V-\eta)^2 - (\gamma-1)P \right]. \qquad (5.6)$$

VI. Results And Discussion

In this paper we have studied the propagation of plane shock. The case $\mu \le 1$ corresponds to a blast wave problem while $\mu = 0$ gives the problems of uniformly expanding shock wave in a medium with zero temperature gradient. For other value of $\mu = 1 \; to \; 0,$ neither the total energy of the wave is not constant nor does shock wave expand uniformly. The kinematics condition at the inner expanding surface is $V(\overline{\eta}) = \mu$ where $\overline{\eta}$ is the value of η at the inner expanding surface. The kinematics condition demands that the velocity of the fluid particle at the expanding surface is equal to the velocity of the surface it self.

For exhibiting the numerical solution it is convenient to write the field variables in non dimensional form as

$$V(1) = \frac{2}{\gamma+1}, \quad P(1) = 1, \; D(1) = 1.$$

The numerical integration of equations (5.4) – (5.6) is carried out by using Runge-Kutta method for $\gamma^2 = \frac{4}{3}$, $\mu = 0.6, \; 0.8$ and $\beta = 0.4$. Here we take $M_A^{\;2} = 0$ a pure non magnetic case. Thus there is no magnetically dominated layer in the flow field behind the shock ie there is no influence of magnetic forces. The nature of the field variables are illustrated by figures (3.1) – (3.3). From fig. (3.1) and (3.2), we see that velocity and density distribution is minimum at shock front but it increases sharply as we move inwards from shock front. But from fig. (3.3) it is clear that discontinuity in pressure distribution is maximum at shock front and decreases rapidly as we move away from shock front.

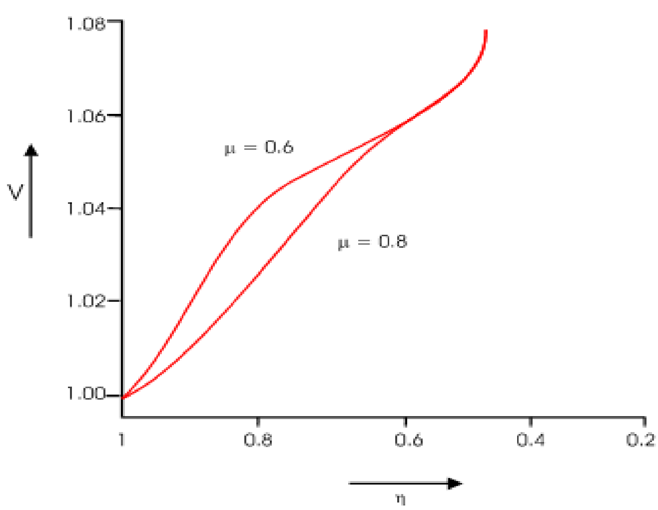

Fig. 3.1
Velocity distribution

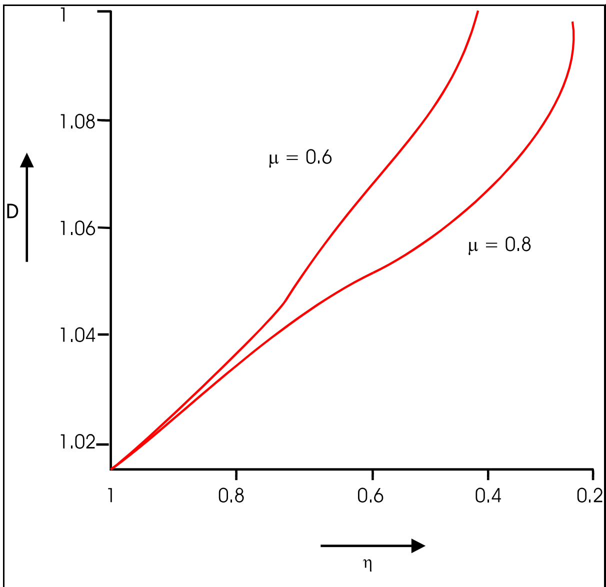

Fig. 3.2
Density distribution

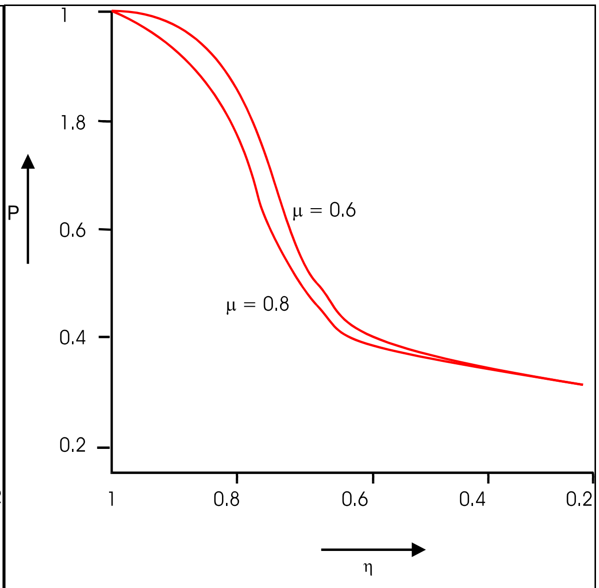

Fig. 3.3
Pressure distribution

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